maximum tangential (swirl) velocities in the flow. Based on their own study of mass injection and the results of other researchers, Mason and Marchman[2] concluded that the introduction of turbulence into the vortex by virtually any means is the dominant factor in tangential velocity reduction and the spreading of the core. The present research seems to substantiate this conclusion.

These results show that vortex dissipation effects similar to those produced by mass injection can be realized without either bleeding power from the engines or using auxiliary power. In fact, using the spinning blade concept, the vortex dissipator may actually be used to produce power. Of course, such results are never free and the effects of these modifications on the wing aerodynamic characteristics as well as the vortex must be studied. The present study seems to indicate that such modifications can significantly reduce the vortex l.azard to trailing aircraft without the use of auxiliary or other power and with few, if any, detrimental effects on the wing aerodynamics.

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Technical Comments

Comment on "Convergence Proof of Discrete-Panel Wing Loading Theories"

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After demonstrating the convergence characteristics of the two-dimensional (2-D) Vortex-Lattice Method (VLM) in a less complete manner than had been done by James[1], DeYoung[2] makes a rather surprising recommendation. He proposes that correction factors be introduced into three-dimensional (3-D) VLM solutions based on the discrepancies between the 2-D VLM pressure distribution and the exact 2-D pressure solution, in spite of the fact that the 2-D VLM solution yields the exact values of lift and moment. The introduction of correction factors that perturb the exact results for lift and moment, and consequently the distribution of shears, bending moments, torques, and hinge moments, must be regarded as a variation of throwing the baby out with the bath water! Moreover, the use of 2-D theoretical results to correct 3-D theoretical results is naive, since experimental results deserve consideration. A technique for adjusting theoretical oscillatory aerodynamic influence coefficients (AIC's) to agree with static wind-tunnel measurements was proposed in 1962 by Rodden and Revell[3] in a survey paper on unsteady AIC's. Since that aspect of the survey has gone largely unnoticed, it is summarized below for the case in which only a limited amount of experimental data for a single deflection mode is available, e.g., the lift curve slope, and the spanwise and/or chordwise location of the aerodynamic center.

Received March 30, 1972.

Index categories: Subsonic and Transonic Flow; Airplane and Component Aerodynamics.

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A premultiplying real diagonal correction matrix† $\lceil W \rfloor$ is introduced to adjust the theoretical AIC's, $[C_{hs}]$ and $[C_h]$ defined in Ref. 4, to yield agreement with experimental data. The corrected static force distribution $\{F_s\}$ becomes

$$\{F_s\} = (qS/\bar{c})\lceil W \rfloor [C_{hs}]\{h\} \tag{1}$$

where q, S, and $\bar{c}$ are the dynamic pressure, reference area, and reference chord, respectively, and $\{h\}$ is the set of deflections of the AIC control points, and the corrected oscillatory force distribution for motion $\{F\}$ becomes

$$\{F\} = \rho\omega^2 b_r{}^2 s \lceil W \rfloor [C_h]\{h\} \tag{2}$$

where ρ, ω, b_r, and s are the density, frequency, reference semichord, and reference semispan, respectively. Equation (2) defines corrected oscillatory AIC's for use in the k method of flutter analysis, as distinguished from Hassig's p-k method of flutter analysis[5] which requires oscillatory AIC's defined as Eq. (1). If the experimental data correspond to a rigid untwisted lifting surface at angle of attack α, we set $\{h\} = \alpha\{x\}$ in Eq. (1) where $\{x\}$ is the set of streamwise coordinates of the AIC control points, and the problem is posed as the solution of

$$\{F_s\} = qS\alpha \lceil W \rfloor [C_{hs}]\{x/\bar{c}\} \tag{3}$$

for $\lceil W \rfloor$ given all the remaining terms in the equation. With only limited experimental data, there are considerably more unknowns than equations and the method of least squares may be employed, although we are now dealing with an underdetermined system of equations rather than the usual overdetermined system. The additional equations are obtained from the requirement that the changes in the theoretical load distribution shall be as uniform as possible or, in least-squares terminology, the weighted sum of the squares of the

† The matrix notation in this Comment employs [ ], ⌈ ⌋, { }, $(\ )^T$, and I to denote rectangular, diagonal, column, transposed, and unit matrices, respectively.

deviations shall be a minimum, where the deviation $\{\epsilon\}$ is defined as the difference between the final and original (unit) correction factors,

$$\{\varepsilon\} = \{W - I\} \tag{4}$$

The weighting function we choose is the theoretical load distribution, $\{F_s^{(t)}\}$ (in which all elements are assumed to be positive), since this choice leads to equal correction elements for the case where only the lift curve slope is known. The weighted least-squares condition then becomes

$$\sum F_s^{(t)}\varepsilon^2 = \{\varepsilon\}^T[F_s^{(t)}]\{\varepsilon\} = \text{a minimum} \tag{5}$$

where the middle matrix factor is a diagonal form of

$$\{F_s^{(t)}\} = qS\alpha[C_{hs}]\{x/\bar{c}\} \tag{6}$$

It is now convenient to make a distinction between the correction factors associated with the number of given experimental conditions and those associated with the remaining least-squares conditions. By analogy with the problem of redundant structures, we refer to the given conditions as basic equations and the corresponding correction factors as basic correction factors W_b, and the additional equations as redundant equations with W_r as the corresponding redundant correction factors. Introducing this distinction into Eqs. (4) and (5) we have

$$\{\varepsilon_b\} = \{W_b - I\} \tag{7}$$

$$\{\varepsilon_r\} = \{W_r - I\} \tag{8}$$

and

$$\{\varepsilon_b\}^T[F_b^{(t)}]\{\varepsilon_b\} + \{\varepsilon_r\}^T[F_r^{(t)}]\{\varepsilon_r\} = \text{a minimum} \tag{9}$$

If we differentiate Eq. (9) with respect to each of the redundant correction factors, we obtain the redundant equations

$$[\partial\varepsilon_b/\partial W_r]^T[F_b^{(t)}]\{\varepsilon_b\} + [\partial\varepsilon_r/\partial W_r]^T[F_r^{(t)}]\{\varepsilon_r\} = 0 \tag{10}$$

where, from Eqs. (7) and (8),

$$[\partial\varepsilon_b/\partial W_r] = [\partial W_b/\partial W_r] \tag{11}$$

$$[\partial\varepsilon_r/\partial W_r] = [I] \tag{12}$$

since W_b and W_r are considered to be the dependent and independent variables, respectively.

We next consider the basic equations. We may write these in the form

$$[Q^{(t)}]\{W\} = \{Q^{(e)}\} \tag{13}$$

or by partitioning

$$[Q_b^{(t)}]\{W_b\} + [Q_r^{(t)}]\{W_r\} = \{Q^{(e)}\} \tag{13}$$

where Q denotes a generalized force (lift, pitching moment, rolling moment, and/or hinge moment) and the elements $Q_{ij}^{(t)}$ of $[Q^{(t)}]$ are the contributions of the ith control point force to the jth generalized force, and $[Q_b^{(t)}]$ is nonsingular.‡ $\{Q^{(e)}\}$ is the given set of experimental conditions to be matched. For example, if the three conditions given are the lift curve slope and aerodynamic center coordinates, then

$$\{Q^{(e)}\} = C_{z_\alpha}\alpha qS \begin{Bmatrix} 1 \\ \bar{x} \\ \bar{y} \end{Bmatrix} \tag{15}$$

The simultaneous solution of Eqs. (10) and (14) is straightforward. From Eq. (14)

$$\{W_b\} = [Q_b^{(t)}]^{-1}(\{Q^{(e)}\} - [Q_r^{(t)}]\{W_r\}) \tag{16}$$

from which, by differentiation,

$$[\partial W_b/\partial W_r] = -[Q_b^{(t)}]^{-1}[Q_r^{(t)}] \tag{17}$$

Then, combining Eqs. (7, 8, 11, 12, and 16) into Eq. (10) permits solution for the redundant weighting factors,

$$\{W_r\} = [A]^{-1}\{B\} \tag{18}$$

where

$$[A] = [F_r^{(t)}] + [\partial W_b/\partial W_r]^T[F_b^{(t)}][\partial W_b/\partial W_r] \tag{19}$$

and

$$\{B\} = [\partial W_b/\partial W_r]^T\{F_b^{(t)}\} + \{F_r^{(t)}\} - [\partial W_b/\partial W_r]^T\{F_b^{(t)}\}[Q_b^{(t)}]^{-1}\{Q^{(e)}\} \tag{20}$$

in which the derivative matrix is given in Eq. (17). The basic correction factors are found by substituting Eq. (18) back into Eq. (16), and the correction matrix is finally found by properly assembling the basic and redundant correction factors into the diagonal format.

The choice of a premultiplying real diagonal correction matrix was obviously arbitrary and the format was selected for its simplicity. Reference 6 has suggested the alternative of a postmultiplying correction matrix but the reason for the preference is not indicated. A postmultiplier may be regarded as a correction for thickness effects and/or for camber induced by boundary-layer displacement effects, and can be derived in a manner similar to the foregoing procedure for the premultiplier in the case of limited data; Ref. 6 assumes as many experimental conditions as there are unknown correction factors. Finally, we note that the correction of the oscillatory AIC's in Eq. (2) contains the additional assumption that the correction factors are independent of the reduced frequency. Bergh and Zwaan[7] have shown that this is a good assumption when the experimental distribution is described reasonably well by theory.

It has been shown that a correction matrix can be defined which provides a means of unifying the use of theoretical AIC's and experimental data for a single deflection mode.§ The result of this unification is that the empirically modified matrices of static or oscillatory AIC's retain the generality of the original theoretical foundation while including the experimental results as limiting values, and permit the calculation of static or oscillatory load distributions for arbitrary deformation modes.

‡ The selection of the AIC control points to which the basic correction factors correspond is analogous to the choice of statically determinate forces in the analysis of redundant elastic structures by the Redundant Force Method. If there are very few experimental conditions the choice is made by inspection choosing non-colinear points to permit matching moments about different axes. If there are more than a few, randomly chosen points in non-colinear groups have been found satisfactory. In any event, $[Q^{(t)}]$ can always be partitioned into $[Q_r^{(t)}]$ and the non-singular $[Q_b^{(t)}]$ by Gaussian elimination.

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Reply by Author to W. P. Rodden

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THE comment by Rodden is a grossly unrealistic reading of Ref. 1. The following two paragraphs are quoted from this reference.

"Comparing with thin-airfoil theory, the lift-curve slope is given exactly for any N by twice the averaged value of $\gamma_n/V\alpha$. Thus, for $N=3$, $c_{1\alpha}=\frac{2}{3}[(15\pi/8)+(3\pi/4)+(3\pi/8)]=2\pi$. Likewise the moment is given exactly for any N. Besides for the chordwise cotangent loading, the lift and moment are summed exactly for the first and second sine harmonics of chordwise loading.

The discrete chordwise loading terms γ_n/V are constant over a given incremental chord distance c/N. For plotting the chordwise loading, γ_n/V is positioned at the quarter-chord of the nth segment, that is at $\xi_n=(n-\frac{3}{4})/N$. A chordwise loading factor can be formed that relates γ/V at ξ_n with γ_n/V. This factor defined by f_n is the ratio of thin-airfoil theory value, Eq. (14), to the incremental loading theory value, Eq. (9)."

Much of the objective of Ref. 1 is to provide a simple, short, rigorous analysis to prove that the chordwise loading is integrated exactly for any N-paneled lattice. The first quoted paragraph shows that this $2N$ line theory integrates exactly as the simpler 2 line theory ($\frac{1}{4}-\frac{3}{4}$ – chord wing theory), which applies for all aspect ratios.

The second quoted paragraph concerns the problem when attempts are made to plot the chordwise loading. Although the sum of the panel elemental vortex lifts ($2\gamma_n c/N$) sum up to the exact total chord lift, the chord loading γ_n is not necessarily the exact γ at chord station ξ_n. In the second quoted paragraph a loading factor, f_n, is defined which aids in plotting the chordwise loading distribution. As repeated several times in Ref. 1, for plotting, the chordwise loading at ξ_n is defined by using the chordwise loading factor as in $\gamma/V=f_n\gamma_n/V$. Qualitatively, this f_n factor is independent of aspect ratio. Slender wing theory shows that the leading edge loading stays as high as that resulting from two-dimensional planar thin wing theory.

I do not throw babies out with the bath water, they are too precious.

Received May 4, 1972.
Index categories: Subsonic and Transonic Flow; Airplane and Component Aerodynamics.

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